

Study Guide for the Final Exam

The Final Exam is a comprehensive, departmental exam. It will consist of 40 multiple choice questions. Please use this guide in conjunction with the Midterm Study Guide.

Section 10.1 Roots and Radicals

1. Evaluate the n th root of a number.

The n th root of the number is the number that must be raised to the n th power to get back to the original number.

$$\sqrt[n]{a} = b \text{ if } b^n = a$$

Some examples: $\sqrt{81} = 9$, $\sqrt[3]{-8} = -2$, $\sqrt[4]{625} = 5$, $\sqrt[5]{32} = 2$

2. Determine if the root of a number is rational, irrational or not a real number.

If the index (value of n) is even and a is negative, then $\sqrt[n]{a}$ is not a real number.

Examples: $\sqrt{-4}$, $\sqrt{-100}$, $\sqrt[4]{-16}$

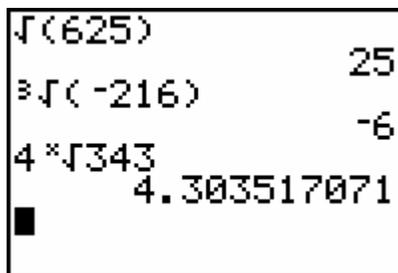
If the radicand (value of a) is a perfect power of the index, then $\sqrt[n]{a}$ is a rational number.

Examples: $\sqrt{25}$, $\sqrt[3]{27}$, $\sqrt[4]{81}$, $\sqrt{\frac{49}{64}}$

If the radicand is not a perfect power of the index and the root is a real number, then $\sqrt[n]{a}$ is irrational.

Examples: $\sqrt{5}$, $\sqrt[3]{12}$, $\sqrt[4]{8}$

3. Use your calculator to find roots.



Keystrokes on calculator:

2^{nd} , x^2 , 625,), Enter

MATH, 4, 216,), Enter

4, MATH, 5, 343, Enter

(The 4 represents the index. If the index is greater than 3, you must type this first before you select the root key.)

4. Apply the inverse properties of roots.

$$\left(\sqrt[n]{a}\right)^n = a, \text{ if } \sqrt[n]{a} \text{ is defined.}$$

Examples:

$$\left(\sqrt{5}\right)^2 = 5$$

$$\left(\sqrt{x+1}\right)^2 = x+1 \text{ (provided } x+1 \geq 0\text{)}$$

$$\left(\sqrt[3]{-8}\right)^3 = -8$$

Section 10.2 Rational Exponents

1. Evaluate exponential expressions with fractional exponents.

$$x^{1/n} = \sqrt[n]{x} \qquad x^{m/n} = \left(\sqrt[n]{x}\right)^m$$

Examples:

$$-25^{1/2} = -\sqrt{25} = -5 \text{ (The negative sign is not connected to the base of the } \frac{1}{2} \text{ power.)}$$

$$(-16)^{1/2} = \sqrt{-16} \text{ not a real number}$$

$$81^{3/4} = \left(\sqrt[4]{81}\right)^3 = (3)^3 = 27$$

$$16^{5/4} = \left(\sqrt[4]{16}\right)^5 = (2)^5 = 32$$

$$8^{-2/3} = \frac{1}{8^{2/3}} = \frac{1}{\left(\sqrt[3]{8}\right)^2} = \frac{1}{2^2} = \frac{1}{4}$$

2. Use the properties of exponents to simplify exponential expressions.

$$a^m \cdot a^n = a^{m+n} \qquad \left(a^m\right)^n = a^{mn} \qquad (ab)^m = a^m b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m} \quad (b \neq 0) \qquad \frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$$

Examples:

$$9^{4/5} \cdot 9^{1/5} = 9^{5/5} = 9^1 = 9$$

$$(16x^8)^{1/4} = 16^{1/4} (x^8)^{1/4} = 2x^2$$

$$\frac{100^{9/4}}{100^{7/4}} = 100^{(9/4-7/4)} = 100^{2/4} = 100^{1/2} = \sqrt{100} = 10$$

Section 10.3 Simplifying Radicals

1. Use the product rule to multiply radicals.

$$\sqrt[n]{x} \cdot \sqrt[n]{y} = \sqrt[n]{xy}$$

Examples:

$$\sqrt{3} \cdot \sqrt{5} = \sqrt{3 \cdot 5} = \sqrt{15}$$

$$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$$

$$\sqrt{3x} \cdot \sqrt{3x} = \sqrt{9x^2} = 3x$$

2. Use the quotient rule to divide radicals.

$$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$$

Examples:

$$\frac{\sqrt{18}}{\sqrt{2}} = \sqrt{\frac{18}{2}} = \sqrt{9} = 3$$

$$\frac{20\sqrt{10}}{4\sqrt{5}} = \frac{20}{4} \cdot \sqrt{\frac{10}{5}} = 5\sqrt{2}$$

3. Use the product rule to simplify radicals.

Remove any factors that are perfect powers of the index from the radicand. Rewrite the radicand as a product of this perfect power and the remaining factor, if possible. Apply the product rule to simplify the result.

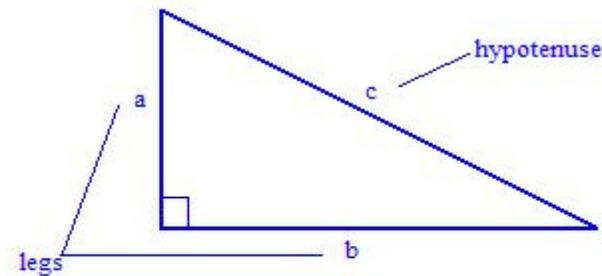
Examples:

$$\sqrt{120} = \sqrt{4 \cdot 30} = \sqrt{4} \cdot \sqrt{30} = 2\sqrt{30}$$

$$\sqrt[3]{40} = \sqrt[3]{8 \cdot 5} = \sqrt[3]{8} \cdot \sqrt[3]{5} = 2\sqrt[3]{5}$$

$$\sqrt{18x^5} = \sqrt{9 \cdot 2x^4x} = \sqrt{9x^4} \sqrt{2x} = 3x^2 \sqrt{2x}$$

4. Use the Pythagorean Theorem to find the length of a missing side of a right triangle.



$$a^2 + b^2 = c^2$$

Example: Find the value of b if a = 3 and c = 5.

$$a^2 + b^2 = c^2$$

$$(3)^2 + b^2 = (5)^2$$

$$9 + b^2 = 25$$

$$b^2 = 16$$

$$b = 4$$

5. Find the distance between two points.

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Example: Find the exact distance between the points (6, -16) and (-20, -2).

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(6 - (-20))^2 + (-16 - (-2))^2}$$

$$d = \sqrt{(6 + 20)^2 + (-16 + 2)^2}$$

$$d = \sqrt{(26)^2 + (-14)^2}$$

$$d = \sqrt{676 + 196}$$

$$d = \sqrt{872}$$

Section 10.4 Adding/Subtracting Radicals

1. Identify like radical terms.

Like radicals have the same index and same radicand.

Examples: $3\sqrt{2}$ and $5\sqrt{2}$ are like radicals but $3\sqrt{2}$ and $5\sqrt[3]{2}$ are not like radicals since their roots are different.

2. Add/subtract radicals.

To add like radicals, combine the coefficients of the radicals and keep the radical part unchanged. Express radicals in simplest radical form before adding.

Examples:

$$\sqrt{6} + \sqrt{6} = 1\sqrt{6} + 1\sqrt{6} = (1+1)\sqrt{6} = 2\sqrt{6}$$

$$3\sqrt{2} + 5\sqrt{2} = (3+5)\sqrt{2} = 8\sqrt{2}$$

$$4\sqrt{5} - \sqrt{5} = 4\sqrt{5} - 1\sqrt{5} = (4-1)\sqrt{5} = 3\sqrt{5}$$

$$3\sqrt{20} + 2\sqrt{45} = 3\sqrt{4 \cdot 5} + 2\sqrt{9 \cdot 5} = 3(2)\sqrt{5} + 2(3)\sqrt{5} = 6\sqrt{5} + 6\sqrt{5} = 12\sqrt{5}$$

$$\sqrt{48} - 5\sqrt{6} = \sqrt{16 \cdot 3} - 5\sqrt{6} = 4\sqrt{3} - 5\sqrt{6} \quad \text{Cannot be combined}$$

Section 10.5 Multiplying/Dividing Radicals

1. Multiply radicals.

To multiply radical expressions, use properties and rules that you have previously learned.

- Product Rule for Radicals $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$ (Any radicals that have the same root (index) can be multiplied. The root stays the same and the radicands are multiplied.)
- Distributive Property: $a(b+c) = ab+ac$
- Multiply two binomials by distributing (FOIL).

$$\boxed{(a+b)(c+d)} = ac + ad + bc + bd$$

Examples:

$$\sqrt{3x} \cdot \sqrt{6x} = \sqrt{18x^2} = \sqrt{9 \cdot 2x^2} = \sqrt{9x^2} \cdot \sqrt{2} = 3x\sqrt{2}$$

$$\sqrt[3]{4}(\sqrt[3]{2} - 4) = \sqrt[3]{8} - 4\sqrt[3]{4} = 2 - 4\sqrt[3]{4}$$

$$(\sqrt{5} + 6)(\sqrt{5} - 6) = \sqrt{25} - 6\sqrt{5} + 6\sqrt{5} - 36 = 5 - 36 = -31$$

$$(4 + \sqrt{10})^2 = (4 + \sqrt{10})(4 + \sqrt{10}) = 16 + 4\sqrt{10} + 4\sqrt{10} + \sqrt{100} = 16 + 8\sqrt{10} + 10 = 26 + 8\sqrt{10}$$

2. Divide radicals – really just know how to reduce fractions with radicals.

Reduce any radicals prior to reducing the fraction.

$$\frac{4 + \sqrt{32}}{4} = \frac{4 + \sqrt{16 \cdot 2}}{4} = \frac{4 + 4\sqrt{2}}{4} = \frac{4}{4} + \frac{4\sqrt{2}}{4} = 1 + \sqrt{2}$$

$$\begin{aligned} \frac{10y + \sqrt{54y^3}}{5y} &= \frac{10y + \sqrt{9 \cdot 6y^2 \cdot y}}{5y} \\ &= \frac{10y + 3y\sqrt{6y}}{5y} \\ &= \frac{y(10 + 3\sqrt{6y})}{5y} \\ &= \frac{10 + 3\sqrt{6y}}{5} \end{aligned}$$

Section 10.6 Solving Radical Equations

1. Solve a radical equation. (Remember to check!)

The square root expression must be isolated to one side of the equation. Square both sides of the equation. Solve the resulting equation and check your answers.

$$1) \quad \sqrt{x+2} = 3$$

$$(\sqrt{x+2})^2 = (3)^2 \quad (\text{Radical is on a side by itself, so square both sides.})$$

$$x+2 = 9 \quad (\text{Result of squaring})$$

$$x+2-2 = 9-2 \quad (\text{Subtract 2 from both sides.})$$

$$x = 7$$

Check:

$$\sqrt{7+2} = 3$$

$$\sqrt{9} = 3$$

$$3 = 3 \quad \text{True}$$

So, $x = 7$ is the solution to the equation.

$$2) \quad \sqrt{2x+1} + 4 = 3$$

$$\sqrt{2x+1} + 4 - 4 = 3 - 4 \quad (\text{Subtract 4 from both sides to isolate radical.})$$

$$\sqrt{2x+1} = -1$$

$$(\sqrt{2x+1})^2 = (-1)^2 \quad (\text{Square both sides to remove the radical.})$$

$$2x+1 = 1 \quad (\text{Result of squaring})$$

$$2x = 0 \quad (\text{Subtract 1 from both sides.})$$

$$x = 0 \quad (\text{Divide both sides by 2.})$$

Check:

$$\sqrt{2(0)+1} + 4 = 3$$

$$\sqrt{0+1} + 4 = 3$$

$$\sqrt{1} + 4 = 3$$

$$1 + 4 = 3$$

$$5 = 3 \quad \text{False, so } x = 0 \text{ is not a solution to the equation.}$$

So, there is no solution to this equation.

$$\begin{aligned}
3) \quad & \sqrt{2x+1} = x-1 \\
& (\sqrt{2x+1})^2 = (x-1)^2 \\
& 2x+1 = x^2 - 2x+1 \\
& 0 = x^2 - 4x \\
& 0 = x(x-4) \\
& x = 0 \text{ or } x = 4
\end{aligned}$$

Check:

$$\begin{aligned}
& x = 0 \\
& \sqrt{2(0)+1} = 0-1 \\
& \sqrt{0+1} = -1 \\
& \sqrt{1} = -1 \\
& 1 = -1 \quad \text{False, so } x = 0 \text{ is not a solution.}
\end{aligned}$$

$$\begin{aligned}
& x = 4 \\
& \sqrt{2(4)+1} = 4-1 \\
& \sqrt{8+1} = 3 \\
& \sqrt{9} = 3 \\
& 3 = 3 \quad \text{True, so } x = 4 \text{ is a solution to the equation.}
\end{aligned}$$

The solution set to this equation is $\{4\}$.

Section 10.7 Complex Numbers

1. Know the definition of i and i^2 .

$$\begin{aligned}
i &= \sqrt{-1} \\
i^2 &= -1
\end{aligned}$$

2. Write non-real numbers in complex form.

$$\sqrt{-k} = i\sqrt{k}$$

Examples:

$$\sqrt{-16} = \sqrt{16 \cdot -1} = \sqrt{16} \cdot \sqrt{-1} = 4i$$

$$\sqrt{-9} = \sqrt{9 \cdot -1} = \sqrt{9} \cdot \sqrt{-1} = 3i$$

$$\sqrt{-4} = \sqrt{4 \cdot -1} = \sqrt{4} \cdot \sqrt{-1} = 2i$$

$$\sqrt{-7} = i\sqrt{7}$$

$$\sqrt{-175} = \sqrt{-25 \cdot 7} = \sqrt{-25} \cdot \sqrt{7} = 5i\sqrt{7}$$

$$5 - \sqrt{-36} = 5 - 6i$$

3. Perform operations with complex numbers.

Add/Subtract Complex Numbers:

$$(-2 + 10i) + (5 - 5i) = -2 + 10i + 5 - 5i = -2 + 5 + 10i - 5i = 3 + 5i$$

$$(7 + 3i) - (2 - i) = (7 + 3i) - 1(2 - i) = 7 + 3i - 2 + i = 7 - 2 + 3i + i = 5 + 4i$$

Multiply Complex Numbers:

$$\sqrt{-25} \cdot \sqrt{-64} = 5i \cdot 8i = 40i^2 = 40(-1) = -40$$

$$4i \cdot 8i = 32i^2 = 32(-1) = -32$$

$$3i(-7 + 4i)$$

$$-21i + 12i^2$$

$$-21i + 12(-1)$$

$$-21i - 12$$

$$-12 - 21i$$

$$(3 + 7i)(9 - 4i)$$

$$27 - 12i + 63i - 28i^2$$

$$27 + 51i - 28(-1)$$

$$27 + 51i + 28$$

$$55 + 51i$$

$$(6 + 8i)^2$$

$$(6 + 8i)(6 + 8i)$$

$$36 + 48i + 48i + 64i^2$$

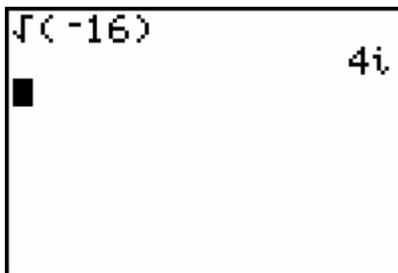
$$36 + 96i + 64(-1)$$

$$36 + 96i - 64$$

$$-28 + 96i$$

4. Use the calculator to check.

To write numbers as complex numbers, change mode to a + bi.



You can also perform operations with complex numbers on your calculator. To enter the complex number i , press 2^{nd} and decimal point.

Here are some examples. These problems come from examples worked above by hand.

$$\begin{array}{r}
 (-2+10i)+(5-5i) \\
 (7+3i)-(2-i) \\
 \sqrt{(-25)}*\sqrt{(-64)} \\
 \blacksquare
 \end{array}$$

$$\begin{array}{r}
 4i*8i \\
 3i(-7+4i) \\
 (3+7i)(9-4i) \\
 \blacksquare
 \end{array}$$

$$\begin{array}{r}
 (6+8i)^2 \\
 \sqrt{(-27)}/\sqrt{(3)} \\
 \blacksquare
 \end{array}$$

Section 11.1 Square Root Property

1. Solve an equation using the Square Root Property.

The Square Root Property states that if

$$\begin{array}{l}
 x^2 = k \\
 x = \sqrt{k} \text{ or } x = -\sqrt{k}
 \end{array}$$

Examples:

$$\begin{array}{ll}
 x^2 = 81 & x^2 = -16 \\
 x = \sqrt{81} \text{ or } x = -\sqrt{81} & x = \sqrt{-16} \text{ or } x = -\sqrt{-16} \\
 x = 9 \quad x = -9 & x = 4i \quad x = -4i
 \end{array}$$

$$\begin{array}{l}
 (x+4)^2 = 5 \\
 x+4 = \sqrt{5} \text{ or } x+4 = -\sqrt{5} \\
 x = -4 + \sqrt{5} \quad x = -4 - \sqrt{5}
 \end{array}$$

Section 11.2 Completing the Square

1. Find the number that completes the square for a given binomial of the form $x^2 + bx$.

To complete the square means to find the number that must be added to a binomial to make it a perfect square trinomial. If the binomial is of the form $x^2 + bx$, then the number that completes the square is $\left(\frac{b}{2}\right)^2$; that is, half of the coefficient of x , squared.

Example: Find the number that completes the square of $x^2 + 22x$.

The number that completes the square is $\left(\frac{22}{2}\right)^2 = (11)^2 = 121$.

2. Solve a quadratic equation by completing the square.

This method is really not the method of choice. I recommend only using it if the equation has a coefficient of 1 on the squared term and the coefficient of x is even.

Example:

$$x^2 - 8x + 5 = 0$$

$$x^2 - 8x = -5$$

$$x^2 - 8x + 16 = -5 + 16$$

$$(x - 4)^2 = 11$$

$$x - 4 = \sqrt{11} \text{ or } x - 4 = -\sqrt{11}$$

$$x = 4 + \sqrt{11} \text{ or } x = 4 - \sqrt{11}$$

Section 11.3 Quadratic Formula

1. Solve a quadratic equation using the quadratic formula.

If a quadratic equation is of the form $ax^2 + bx + c = 0$ ($a \neq 0$), then the solutions to the equation are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example:

$$3x(x-5) = 4$$

$$3x^2 - 15x = 4$$

$$3x^2 - 15x - 4 = 0$$

$$a = 3, b = -15, c = -4$$

$$x = \frac{-(-15) \pm \sqrt{(-15)^2 - 4(3)(-4)}}{2(3)}$$

$$x = \frac{15 \pm \sqrt{225 + 48}}{6}$$

$$x = \frac{15 \pm \sqrt{273}}{6}$$

2. Find the discriminant and know what this number tells you about the solutions to the equation.

The discriminant is the number in the radical of the quadratic formula, that is, $b^2 - 4ac$.

| $b^2 - 4ac$ | Conclusion |
|-----------------------------------|---|
| positive and perfect square | two different rational solutions; equation is factorable |
| positive and not a perfect square | two different irrational solutions |
| zero | one rational solution |
| Negative | two different non-real complex solutions |

Example: Find the discriminant of $4x^2 - 5x - 1 = 0$ and determine the types of solutions.

The discriminant is $b^2 - 4ac = (-5)^2 - 4(4)(-1) = 25 + 16 = 41$. Since this is a positive number that is not a perfect square, the equation has two irrational solutions.

3. Use your calculator to check your solutions and to find approximations of irrational solutions.

Please see my notes that are posted in MML Study Notes on this topic. These notes are very detailed.

Section 11.4 Equations Quadratic in Form

1. Solve a rational equation.

To solve a rational equation, clear the fractions by multiplying both sides by the LCD. Solve the resulting equation. Exclude any solutions that make the denominator zero.

Example:

$$1 - \frac{4}{x} - \frac{12}{x^2} = 0$$

$$x^2 \left(1 - \frac{4}{x} - \frac{12}{x^2} \right) = x^2(0)$$

$$x^2 - 4x - 12 = 0$$

$$(x - 6)(x + 2) = 0$$

$$x - 6 = 0 \text{ or } x + 2 = 0$$

$$x = 6 \qquad x = -2$$

The solution set is $\{-2, 6\}$.

2. Solve a square root equation.

This is a review of section 10.6. Please see these notes for an example.

3. Solve a higher degree factorable equation.

A higher degree equation has a degree greater than 2. Start solving this equation by writing it in standard form and then factoring.

Example:

$$x^4 - 3x^2 - 4 = 0$$

$$(x^2 - 4)(x^2 + 1) = 0$$

$$x^2 - 4 = 0 \text{ or } x^2 + 1 = 0$$

$$x^2 = 4 \quad \text{or} \quad x^2 = -1$$

$$x = \pm 2 \qquad x = \pm\sqrt{-1} = \pm i$$

The solution set is $\{-2, 2, i, -i\}$

Section 11.5 Applications

1. Solve applications using the Pythagorean Theorem.

Example: Find the lengths of the sides of a right triangle in which the longer leg is one inch more than the shorter leg and the hypotenuse is two inches more than the shorter leg.

Let x = length of the shorter leg
 $x + 1$ = length of the longer leg
 $x + 2$ = length of the hypotenuse

$$x^2 + (x+1)^2 = (x+2)^2$$

$$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$$

$$2x^2 + 2x + 1 = x^2 + 4x + 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$x = 3 \text{ or } x = -1$$

The lengths of the sides of the right triangle are 3, 4, and 5 inches.

2. Solve an application involving area.

Example: Find the length and width of a rectangle whose area is 27 square feet and whose length is 6 feet less than the width.

Let w = width
 $w-6$ = length

$$w(w-6) = 27$$

$$w^2 - 6w - 27 = 0$$

$$(w-9)(w+3) = 0$$

$$w = 9 \text{ or } w = -3$$

The width is 9 feet and the length is $9 - 6 = 3$ feet.

3. Solve an application given a model.

Example: The height of a ball thrown from the ground can be given by $h = -16t^2 + 32t$, where t is the number of seconds after the ball is thrown. Find the time that the ball reaches a height of 16 feet.

$$-16t^2 + 32t = 16$$

$$-16t^2 + 32t - 16 = 0$$

$$t^2 - 2t + 1 = 0$$

Divide both sides by the GCF of -16.

$$(t-1)(t-1) = 0$$

$$t = 1$$

The ball will reach a height of 16 feet in 1 second after it is thrown.

Section 11.6 Graphing Quadratic Equations

1. Graph a parabola by identifying the vertex and two other points.

If an equation is of the form $y = a(x-h)^2 + k$, the vertex of the form is (h, k) . Find two additional points – one on each side of the vertex to get the graph.

Example: Graph the equation $y = 2(x+3)^2 - 8$.

- The vertex of the parabola is $(-3, -8)$.
- Since $a = 2$, the parabola opens up.
- Find two additional points. You can do this by hand or on the calculator.

By hand: Let $x = -5$ and $x = -1$ (a value to the left of -3 and to the right of -3)

$$x = -5: y = 2(-5+3)^2 - 8 = 2(-2)^2 - 8 = 2(4) - 8 = 8 - 8 = 0 \quad (-5, 0)$$

$$x = -1: y = 2(-1+3)^2 - 8 = 2(2)^2 - 8 = 2(4) - 8 = 8 - 8 = 0 \quad (-1, 0)$$

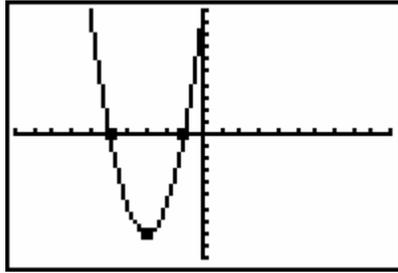
By the calculator:

Plot1 Plot2 Plot3
Y1 = 2(X+3)^2 - 8
Y2 =
Y3 =
Y4 =
Y5 =
Y6 =
Y7 =

| X | Y1 |
|----|----|
| -6 | 10 |
| -5 | 0 |
| -4 | -6 |
| -3 | -8 |
| -2 | -6 |
| -1 | 0 |
| 0 | 10 |

X = -6

- Plot the three points and draw the parabola.



Section 11.8 Nonlinear Inequalities

1. Solve a quadratic inequality.

Example: Solve $x^2 - x - 6 > 0$.

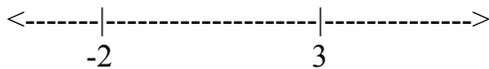
Solution:

$$x^2 - x - 6 = 0 \quad \text{Find solutions to the associated equation.}$$

$$(x - 3)(x + 2) = 0$$

$$x = 3 \text{ or } x = -2$$

Plot the solutions on the number line.



Test points from the regions on the number line:

$$x = -3: (-3)^2 - (-3) - 6 > 0 \Rightarrow 6 > 0 \text{ True}$$

$$x = 0: (0)^2 - (0) - 6 > 0 \Rightarrow -6 > 0 \text{ False}$$

$$x = 4: (4)^2 - (4) - 6 > 0 \Rightarrow 6 > 0 \text{ True}$$

Shade the regions that test true and put the correct symbol on the endpoints.



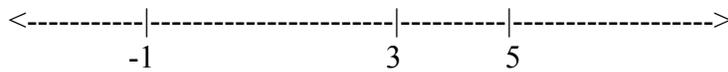
The interval for the solution is $(-\infty, -2) \cup (3, \infty)$.

2. Solve a polynomial inequality.

Example: Solve $(x-5)(x+1)(x-3) \leq 0$.

Solution: The solutions to the equation are $x = 5$, $x = -1$, $x = 3$.

Plot these numbers on the number line and test points from each of the regions formed.



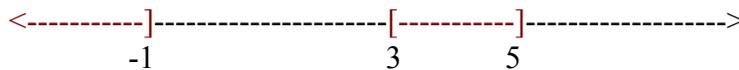
$$x = -2: (-2-5)(-2+1)(-2-3) \leq 0 \Rightarrow (-7)(-1)(-5) \leq 0 \Rightarrow -35 \leq 0 \text{ True}$$

$$x = 0: (0-5)(0+1)(0-3) \leq 0 \Rightarrow (-5)(1)(-3) \leq 0 \Rightarrow 15 \leq 0 \text{ False}$$

$$x = 4: (4-5)(4+1)(4-3) \leq 0 \Rightarrow (-1)(5)(1) \leq 0 \Rightarrow -5 \leq 0 \text{ True}$$

$$x = 6: (6-5)(6+1)(6-3) \leq 0 \Rightarrow (1)(7)(3) \leq 0 \Rightarrow 21 \leq 0 \text{ False}$$

Shade the regions that tested true and include the appropriate symbol on the endpoints.



Solution: $(-\infty, -1] \cup [3, 5]$

Section 9.2 Absolute Value Equations and Inequalities

1. Solve an absolute value equation.

Isolate the absolute value expression to one side of the equation and then use the following property to solve it.

$$|x| = k$$

$$X = k \text{ or } x = -k \quad (\text{if } k \text{ is positive})$$

$$X = k \quad (\text{if } k = 0)$$

No solution if k is negative

Examples:

| | |
|---|--|
| $ x + 2 = 5$ $x + 2 = 5$ or $x + 2 = -5$ $x = 3$ or $x = -7$ | $ 1 - 2x + 4 = 3$ $ 1 - 2x = -1$ (isolate abs. value) |
| Solution set: $\{-7, 3\}$ | No solution |

2. Solve an absolute value inequality.

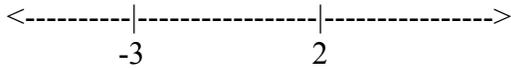
Apply the properties used to solve quadratic and higher degree polynomial inequalities. Find the solutions to the associated equation, plot the points on the number line, and then test a point from each region.

Examples:

$$|2x + 1| > 5$$

$$\text{Solution: } |2x + 1| = 5$$

$$\begin{array}{l} 2x + 1 = 5 \text{ or } 2x + 1 = -5 \\ 2x = 4 \qquad \qquad 2x = -6 \\ x = 2 \qquad \qquad \quad x = -3 \end{array}$$



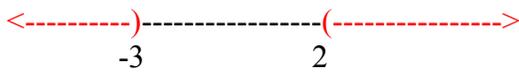
Test points:

$$\begin{array}{l} X = -4: |2(-4) + 1| > 5 \\ \quad \quad \quad |-7| > 5 \\ \quad \quad \quad 7 > 5 \text{ True} \end{array}$$

$$\begin{array}{l} X = 0: |2(0) + 1| > 5 \\ \quad \quad \quad |1| > 5 \\ \quad \quad \quad 1 > 5 \text{ False} \end{array}$$

$$\begin{array}{l} X = 3: |2(3) + 1| > 5 \\ \quad \quad \quad 7 > 5 \text{ True} \end{array}$$

Solution:



Interval: $(-\infty, -3)$ union $(2, \infty)$